Control: Lec 5

Ex

$$GH(s) = \frac{10}{s(1+0.05s)(1+0.25s)}$$

* Draw the bode diagram and find W_{gc}, W_{pc}, GM, PM

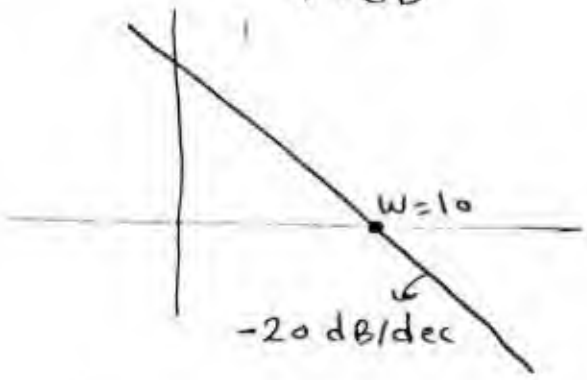
Sol

$$GH(s) = \frac{10}{s\left(1+\frac{s}{20}\right)\left(1+\frac{s}{4}\right)}$$

$s \longrightarrow jw$

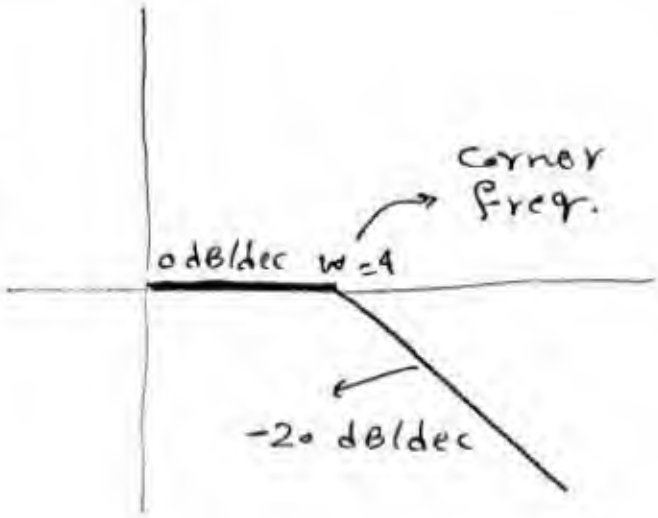
$$\therefore GH(jw) = \frac{10}{jw\left(1+j\frac{w}{20}\right)\left(1+j\frac{w}{4}\right)}$$

Term	$\phi(w)$	11 dB
$\frac{10}{s} \rightarrow \frac{10}{jw}$	-90°	



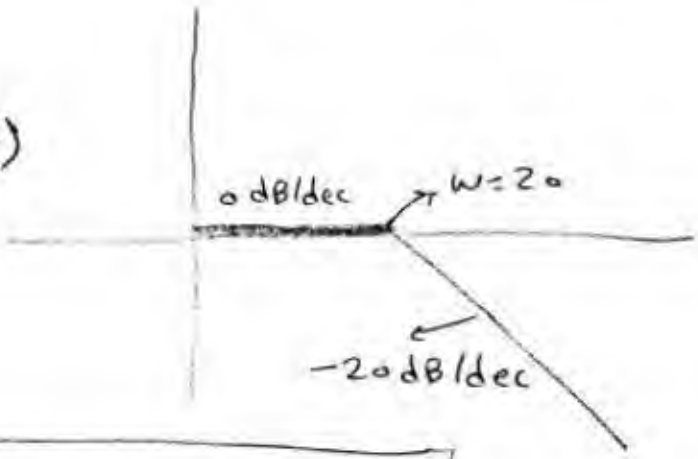
$$\frac{1}{1+\frac{s}{4}} \Downarrow \frac{1}{1+j\frac{\omega}{4}}$$

$$-\tan^{-1}\left(\frac{\omega}{4}\right)$$

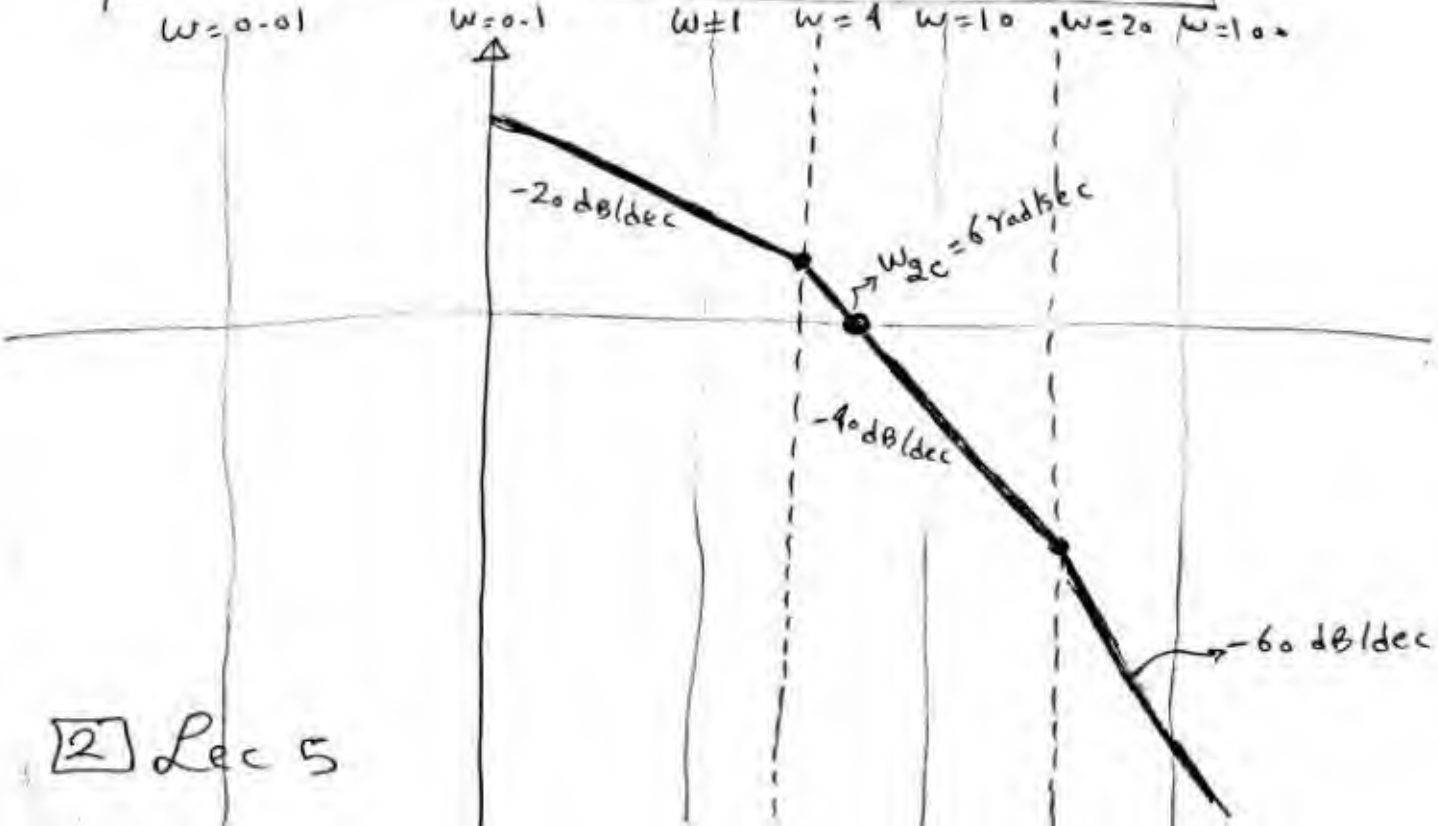


$$\frac{1}{1+\frac{s}{20}} \Downarrow \frac{1}{1+j\frac{\omega}{20}}$$

$$-\tan^{-1}\left(\frac{\omega}{20}\right)$$



$$\phi(\omega) = -90 - \tan^{-1}\left(\frac{\omega}{4}\right) - \tan^{-1}\left(\frac{\omega}{20}\right)$$



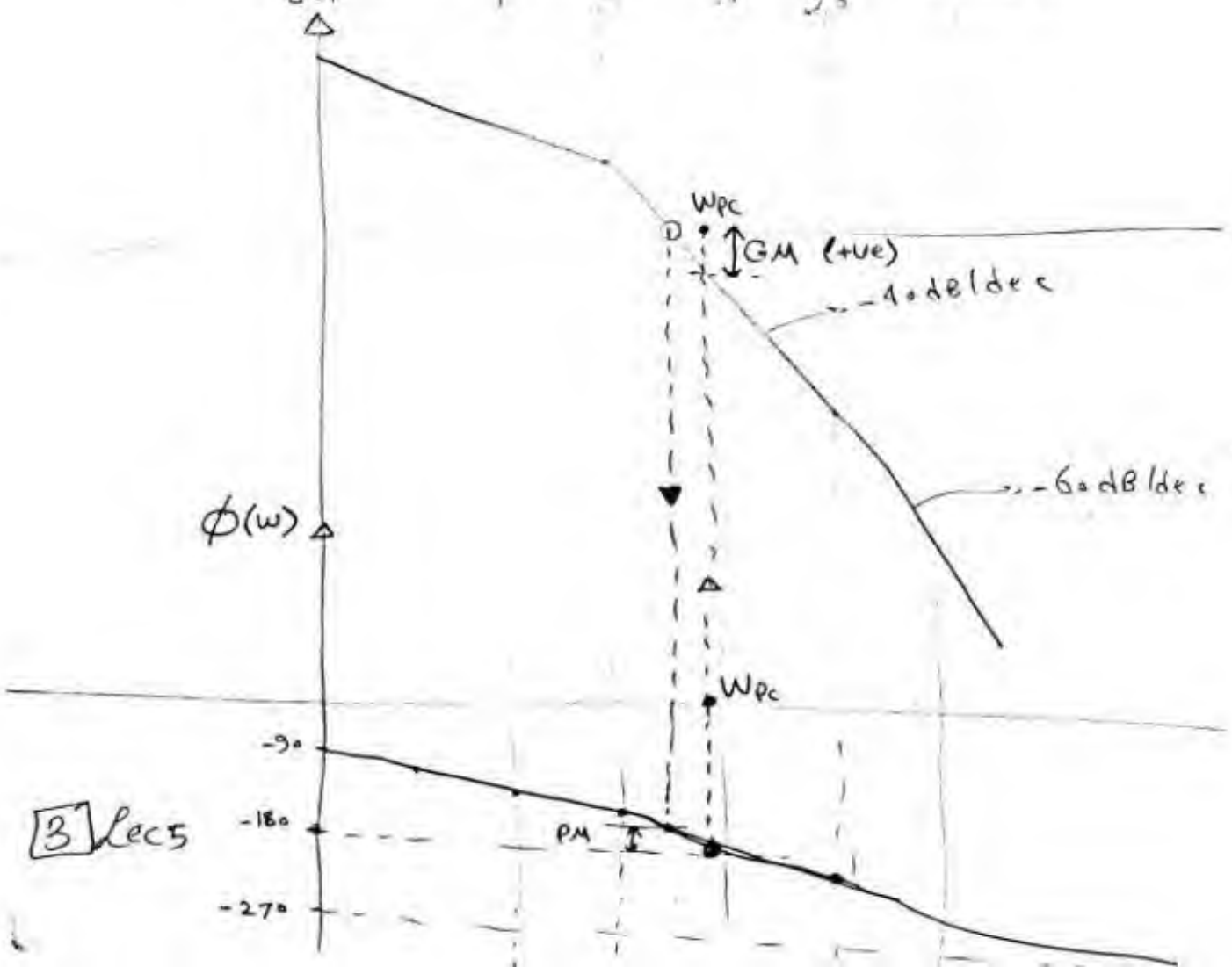
- نعمل جدول لقيم ω لحساب $\phi(\omega)$

ω	0	0.1	0.5	1	$\omega=4$	∞
$\phi(\omega)$	-90	-91.72	-98.5	-106.9	-146.3	-270

ω	6	20
$\phi(\omega)$	-163	-213.7°

$PM = 180 + \phi(\omega_{gc} = 6)$

$= 180 - 163 = 17^\circ$ (+ve) Stable



at $w=9 \Rightarrow \phi(w) = -180 - 26^\circ$

at $w=10 \Rightarrow \phi(w) = -184^\circ$

at $w=8 \Rightarrow \phi(w) = \leftarrow$

← صغوه بال $(w=9)$ لانها أقرب حاجة للزاوية (-180°)

$W_{pc} \simeq 9$ rad/sec

G.M $$G.M = \frac{1}{|G\,H(jw)|_{w = W_{pc} = 9}}$$

$$G.M|_{dB} = 20 \log (G.M)$$

GM
- > 1 stable
- < 1 unstable
- $= 1$ critical

GM_{dB}
- > 0 stable
- $= 0$ critical
- < 0 unstable

$GM|_{dB} = 7$ dB ⟶ at exact

$GM|_{dB} \simeq 3$ dB ⟶ at Approximate

Ex

$$GH(s) = \frac{625(s+1)(s+10)}{s(s+5)(s+50)^2}$$

$$= \frac{(625)(10)\left(1+\frac{s}{1}\right)\left(1+\frac{s}{10}\right)}{(5)(50)^2 s\left(1+\frac{s}{5}\right)\left(1+\frac{s}{50}\right)^2}$$

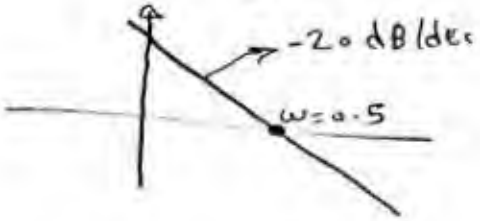
$$= \frac{0.5\left(1+\frac{s}{1}\right)\left(1+\frac{s}{10}\right)}{s\left(1+\frac{s}{5}\right)\left(1+\frac{s}{50}\right)^2}$$

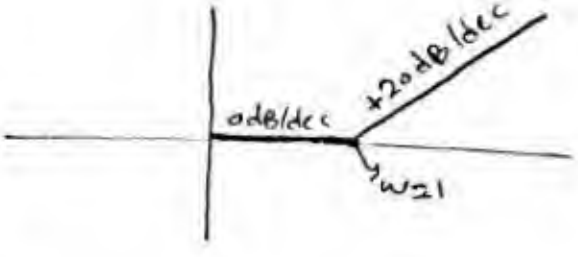
$s \to j\omega$

$$GH(j\omega) = \frac{0.5\left(1+\frac{j\omega}{1}\right)\left(1+j\frac{\omega}{10}\right)}{j\omega\left(1+\frac{j\omega}{5}\right)\left(1+\frac{j\omega}{50}\right)^2}$$

Term	$\phi(\omega)$
$\frac{0.5}{s} \to \frac{0.5}{j\omega}$	-90°
$1+\frac{s}{1}$ ⇓ $1+j\frac{\omega}{1}$	$\tan^{-1}(\omega)$

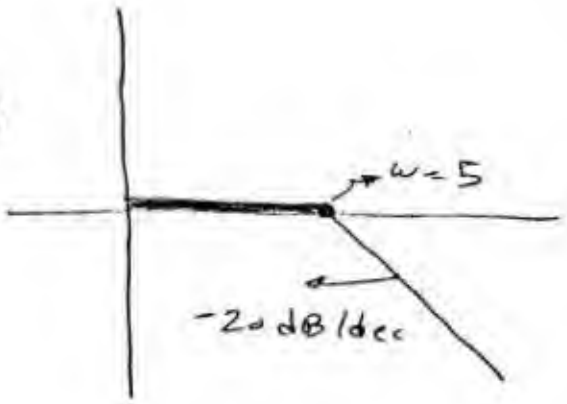
11_{dB}



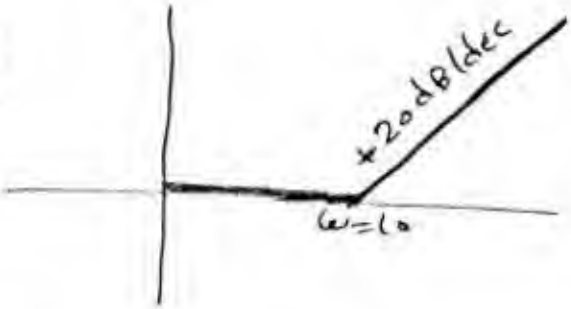


5. $\frac{1}{s}$

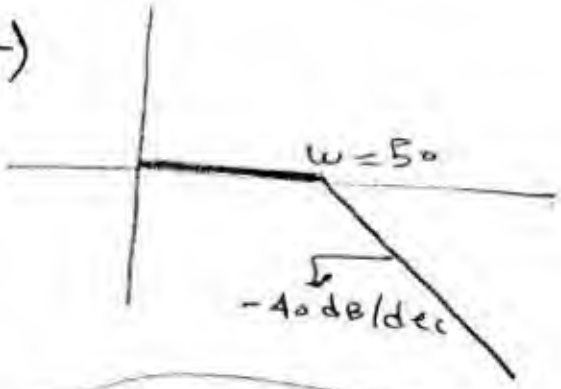
$$\frac{1}{1+\frac{s}{5}} \Rightarrow \frac{1}{1+j\frac{\omega}{5}} \qquad -\tan^{-1}\left(\frac{\omega}{5}\right)$$



$$\left(1+\frac{s}{10}\right) \rightarrow 1+j\frac{\omega}{10} \qquad \tan^{-1}\left(\frac{\omega}{10}\right)$$

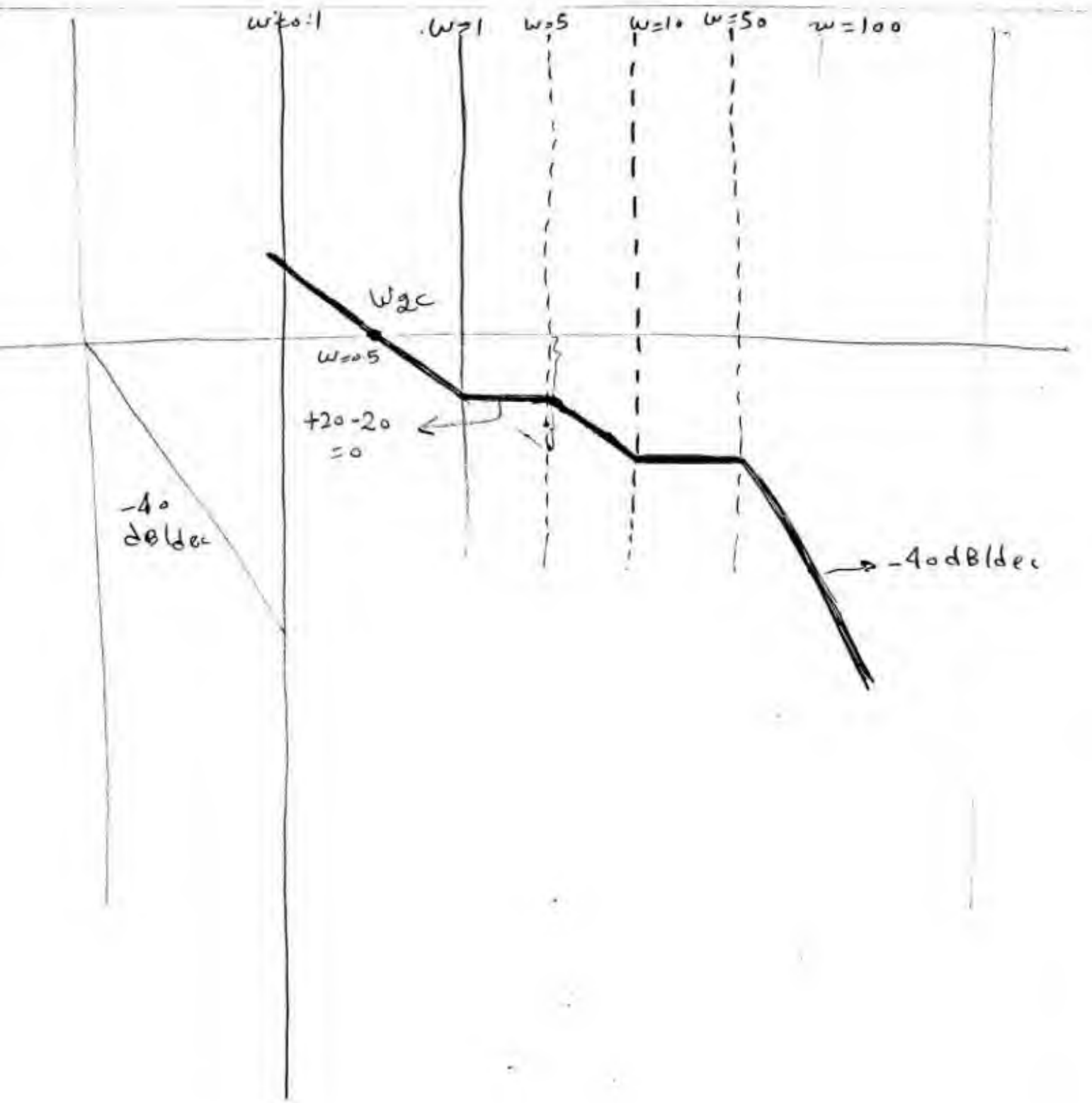


$$\frac{1}{\left(1+\frac{s}{50}\right)^2} \Rightarrow \frac{1}{\left(1+j\frac{\omega}{50}\right)^2} \qquad -2\tan^{-1}\left(\frac{\omega}{50}\right)$$



$$\phi(\omega) = -90 + \tan^{-1}(\omega) + \tan^{-1}\left(\frac{\omega}{10}\right) - \tan^{-1}\left(\frac{\omega}{5}\right) - 2\tan^{-1}\left(\frac{\omega}{50}\right)$$

ω	0		∞
$\phi(\omega)$	-90		-180

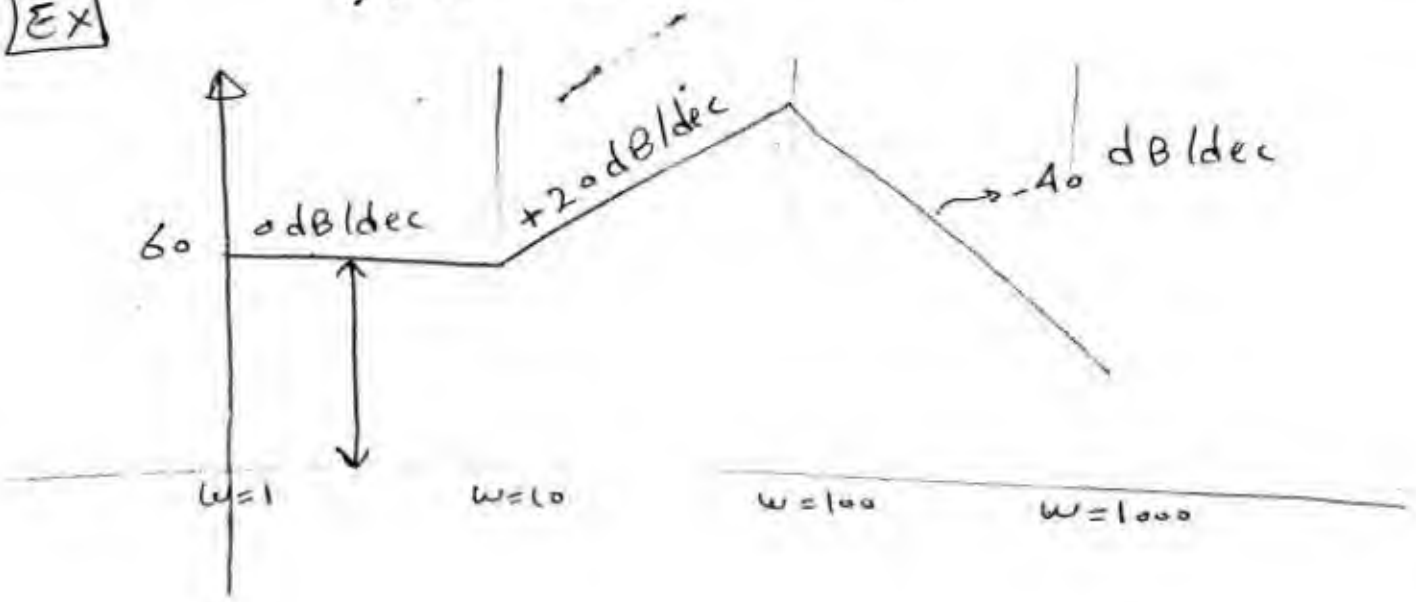

w=0.1
w=1
w=5
w=10
w=50
w=100
wgc
w=0.5
+20-20
=0
-40
dB/dec
-40dB/dec

$$PM = 180^\circ + \phi(\omega = 0.5)$$

$$= +112.57^\circ$$

$$GM = \infty \qquad \text{stable}$$

Ex Find o.L.T.F $GH(s)$



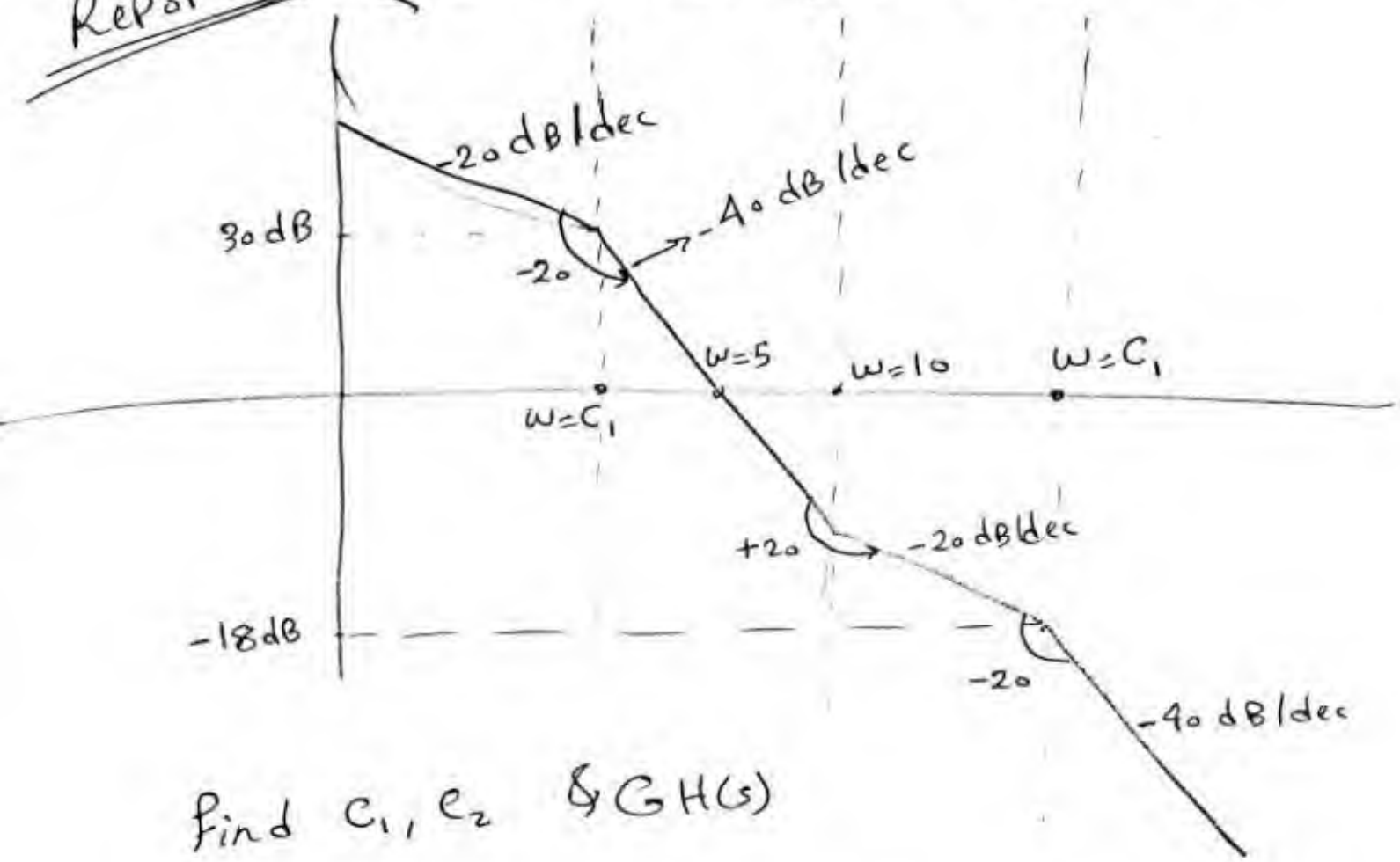
$$GH(s) = \frac{K\left(1+\frac{s}{10}\right)}{\left(1+\frac{s}{100}\right)^3}$$

$$20\log K = 60 \Rightarrow K = 10^3 = 1000$$

$$GH(s) = \frac{1000\left(1+\frac{s}{10}\right)}{\left(1+\frac{s}{100}\right)^3}$$

Sol Prof 5

Report ← يسلم في المحاضرة القادمة



Find C_1, C_2 & $GH(s)$

← لو قيم C_1 ، C_2 معلومة

$$GH(s) = \frac{K\left(1+\frac{s}{10}\right)}{\left(1+\frac{s}{C_1}\right)\left(1+\frac{s}{C_2}\right)}$$

الحل ←

← لو رسمته على ورقه (Semi log) هتعرف تجيب قيم C_1 ، C_2 .

← ممكن تجيبهم بحل رياضي لكنه صعب .